European Research Studies Journal
Volume XXII, Issue 1, 2019

pp. 192-208

Multidimensional Core-peripheral Model

Submitted 25/2/19, 1st revision 28/2/19, accepted 1/3/19

Marcus Gumpert[1]

Abstract:

The following article analyzes why an extension of the core-peripheral model is necessary for realistic economic use. The starting point is the core-peripheral model from Krugman. Various modifications are then made to make the Krugman model more robust, growth-oriented, and realistic.

The models by Ricardian, Heckscher-Ohlin, Krugman, and Solow are combined by the author. This new model provides a general explanation model that is advantageous for the most diverse analyses that is, from rigid to dynamic, and growth theory-oriented model frameworks.

The Ricardian model provides an immobile factor of work, perfect competition, distribution-free productions, comparative cost advantages, and constant economies of scale. The Heckscher-Ohlin theorem provides a second mobile input factor capital and production functions according to the equipment of the regions.

The core-peripheral model provides aspects of monopolistic competition, cost functions, transportation costs, and spatial distribution. Depreciation rates, investment rates, and savings rates are taken over from the Solow model.

We define the following parameters in the new model variant: reasons for model expansion, input factors, technology components, transportation costs, investment rate, savings rate, outputs, number of business/distribution, cost functions, consumer benefits, goods prices, wages, returns, and incomes.

Keywords: *Regional economic disparities, core-peripheral model, growth theoretical model framework, regional policy, multifactorial.*

JEL codes: *R00, R10, R15, R20, O10, O11.*

ISSN: *1108-2976*

[1] *Health Insurance Physicians Unification Saxony, Quality Assurance/Quality Management,* marcus.gumpert@kvsachsen.de

1. Introduction

This article investigate two regions and aims to contribute to the explanation of interregional interactions. Different model frames are used as a starting point: the Ricardian model, Heckscher-Ohlin theorem, core-peripheral model, and Solow model (Gumpert, 2016; Heinemann, 2015; Krugman, 1991a; Krugman, 1991b; Ohlin, 1933; Ramsey, 1928; Ricardo, 1987; Solow, 1956).

The two regions interact with each other, in that region *A* is advanced, capital intensive and industrial, and the region *B* is backward, labor intensive and agricultural. For simplification, full specialization is adopted. To analyze different aspects, a new robust and realistic model is developed by combining the model. The model combination is characterized by the following motivations:

- Rigid input factors linked to a fixed factor
- Expansion by additional mobile factors (e.g., capital)
- Variable forms of economies of scale
- Combination of the specialization mutation of comparative cost advantages and input factor-intensive productions
- Combination of different market forms (e.g., monopolistic or perfect competition)
- Analysis of cost functions with multidimensional input factors as well as fixed, marginal, and variable costs
- Expansion of the utility function to include product variety
- Inclusion of substitution elasticities
- Inclusion of transportation costs for industrial goods - as an analogy to free transportation costs by agricultural goods
- Investment rates and depreciation rates (”*steady state”*)
- Savings rates from the Solow model for direct connection between consumers (*savers*) and companies (*producers*)
- Recording of savings measures within the utility functions
- Further development of the technology component by capital write-downs
- Introduction of a standard unit in the industrial sector to allow for the direct link between the number of companies and individual production quantities of an individual company

From the core-peripheral model, assumptions such as increasing economies of scale, Chamberlain's monopolistic competition, preferences for product diversity -love of variety- (Dixit-Stiglitz), and mobility of factors remain intact (Dixit and Stiglitz, 1977). In addition, individual assumptions of the Ricardian model and Heckscher-Ohlin theorem are accounted for and combined. The parameters for determining the steady state are taken from the Solow model; for example, this applies to the depreciation, investment, and savings rates. The following articles are used as a basis for this: Boschma and Frenken, 2017; Brezis *et al.*, 1993; Brezis and Tsiddon, 1998; Cobb and Douglas, 1928; Desmet and Ortín, 2007; Ehrenfeld, 2004; Gumpert, 2013; Gumpert, 2016; Pindyck and Rubinfeld, 2003; Samuelson and Nordhaus, 2005; Swan, 2004; Venables, 1996).

We define the following parameters in the new model variant: reasons for model expansion, input factors, technology components, transportation costs, depreciation rate, investment rate, savings rate, agricultural output, industrial output, number of business/distribution, cost functions, consumer benefits, specialization pattern, agricultural goods price, industrial goods price, wages, returns, and incomes.

Geographical distribution plays a crucial role in reality, for this reason, it also plays a central role in this work. The observation of a heterogeneous space represents reality nearly. Heterogeneous space has its origin in various triggering criteria: historical events, climatic conditions, geography, or geodesy. This results in the introduction of transportation costs, which include spatial expansion in the model.

Agglomeration effects are also considered. According to Demko, Krugman, Martin, Sunley, and Venables, economic connections lead to inhomogeneous spatial allocations. The market displacement effect and the market expansion effect occur (Demko, 2017; Krugman and Venables, 1990; Krugman and Venables, 1995; Martin and Sunley, 2017; Venables, 1996).

Finally, monopolistic competition should be highlighted. Every company is an individual monopolist and can impose a price premium. Fixed costs, variable costs, and marginal costs are also included. As a result, the expansion of the model leads to increasing economies of scale. On the one hand, reality is characterized by many identical products, which differ slightly in design, execution, etc. On the other hand, cost functions also lead to different product development decisions in companies. These connections and designs make the model robust and applicable in a variety of applications. After all, consumers also want a variety of products. Taking Dixit-Stiglitz into account, various product variants are included in the industrial sector (Dixit and Stiglitz, 1977). This new model is developed in the following section.

2. The model

2.1 Input factors

Different model frames are used as a starting point: the Ricardian model, Heckscher-Ohlin theorem, core-peripheral model, Ramsey model, and Solow model (Fonseca, 2017; Krugman, 1991a; Krugman, 1991b; Ohlin, 1933; Ramsey, 1928; Ricardo, 1987; Solow, 1956).

Consider two regions: A and B. The economy is divided into an industrial region A and an agricultural region B with *. The region A has the advantage of existing industrial production and, therefore, knowledge accumulations. The region B is characterized by an agricultural economy and low labor costs. Region B is defined with a^*. Furthermore, there is one unit of each input factor. Two goods (*agricultural and industrial products*) are analyzed.

$$L = L^* = 1 \tag{1}$$

$$K = K^* = 1 \tag{2}$$

The factor of labor is mobile in the industrial goods sector because the capital goods are not linked to any other aspects. The factor labor is immobile in the industrial products because the immovable third factor soil E is required to produce an agricultural product and this phenomenon depends directly on the factor of work in the agricultural sector. Capital is fully mobile in both sectors and regions. The labor force per sector is marked with L_F and L_F^* for the agricultural sector and L_M and L_M^* for the industrial sector.

The phenomenon of retraining is being developed. Industrial workers can retrain and become active in the agricultural sector. Agricultural workers can retrain and become active in the industrial sector but are always tied to their region and the factor soil *E;* for this reason, there is immobility in the agricultural sector and agricultural produce.

$$L_M = L_M^* = L_F\left(E\right) = L_F^*\left(E^*\right) = 1 \tag{3}$$

$$K_M = K_M^* = K_F = K_F^* = 1 \tag{4}$$

The labor factor is partially mobile between regions. The labor force is mobile in the individual sectors. Factor migration to the region with the highest level of technological progress leads to a balancing of the marginal industrial wages in both regions. There is perfect mobility of industrial workers and part-mobility of agricultural workers.

$$w_M = w_M^* = w_F \tag{5}$$

$$w_M^* = w_M = w_F^* \tag{6}$$

Capital is geographically mobile across both regions and sectors. The capital flows to the region with the highest level of technological progress (Brezis and Tsiddon, 1998). Although each region has one unit of capital, the geographical allocation of the factor is determined by the equality of marginal returns (perfect capital mobility).

$$r_M = r_F = r_M^* = r_F^* \tag{7}$$

2.2 Technology component

In the following production function, a technology component plays a decisive role in the industrial sector in addition to the already defined input factors. Technology component A_M is greater than one and the stronger the learning effect l the higher the output. Through a high level of commitment to the input factors of work and capital, employees are constantly learning more. Learning effect l means that employees can increase the amount of goods produce in a fixed period of time. Furthermore, a higher input quantity of the existing input factors or an increasing number of different input factors leads to a higher output quantity and a higher learning l effect.

$$H_M(t) = \int_0^t Q_M(l)\,dl \tag{8}$$

$$H_M^*(t) = \int_0^t Q_M^*(l)\,dl \tag{9}$$

The technology component in the agricultural sector is one and has no learning effect.

2.3 Transportation costs

Transportation costs T play a decisive role in the model expansion. In the model, costs occur only in the industrial sector to represent differences more optimally. However, it is generally possible to consider transportation costs in both sectors and regions. The transportation costs add a markup to the price of industrial goods.

$$P_{M,T} = T \cdot P_M \tag{10}$$

2.4 Depreciation rate, investment rate, and savings rate

From the Solow model, this publication receives the depreciation rate as and as^*, investment rate i and i^*, investment sum I and I^*, savings rate s and s^*, and savings sum S and S^*. The rates are integrated into the producer side via the capital input factor to achieve the steady state. Integration on the consumer side takes place via the savings rate. Consumers must save part of their total income. The remainder of their income is consumed.

Household savings rates are used directly for corporate investment measures. In this manner, households secure their jobs and make sustainable financial investments. Companies can use the investment measures to absorb, at least, the depreciation loss in the capital sector (Bretschger, 2004; Gärtner, 2006; Solow, 1956).

$$Y = Y \cdot \left(c + s\right) = C + S \tag{11}$$

$$Y^{*} = Y^{*} \cdot \left(c^{*} + s^{*}\right) = C^{*} + S^{*} \tag{12}$$

$$I = i \cdot Y = s \cdot Y = S \tag{13}$$

$$I^{*} = i^{*} \cdot Y^{*} = s^{*} \cdot Y^{*} = S^{*} \tag{14}$$

Equations (15), (16), (17), and (18) show the combinations of the investment rates and depreciation rates with the capital rates.

$$K_{M,SM} = \left(1 + i - as\right) \cdot K_{M} \tag{15}$$

$$K_{M,SM}^{*} = \left(1 + i^{*} - as^{*}\right) \cdot K_{M}^{*} \tag{16}$$

$$K_{F,SM} = \left(1 + i - as\right) \cdot K_{F} \tag{17}$$

$$K_{F,SM}^{*} = \left(1 + i^{*} - as^{*}\right) \cdot K_{F}^{*} \tag{18}$$

In the agricultural sector, the technology component is one; thus, the investment rates are identical to the depreciation rate. In the industrial sector, the technology component A_M can increase, remain the same, or decrease. This result on whether the investment rate is higher, equal, or lower than the depreciation rate in the industrial sector.

$$i \; {}^{\geq}\!\!/\!{}_{<} \; as \tag{19}$$

$$i^{*} = as^{*} \tag{20}$$

2.5 Agricultural output

Production in the agricultural sector is defined by the two input factors. Regarding constant elasticity *γ*, the exponents give a value of one. Due to full competition in the agricultural sector, there is no monopoly premium. Furthermore, the technology A_F^{*} component is one and has no learning effects. There are also no transportation costs. Equation *(21)* shows the output in the agricultural sector in region *B*.

$$Q_{F}^{*} = A_{F}^{*}\left(\square\, 1\right) \cdot \left(\left(1 + i_{F}^{*} - as_{F}^{*}\right) \cdot K_{F}^{*}\right)^{\gamma} \cdot \left(L_{F}^{*}\left(E^{*}\right)\right)^{1-\gamma} \tag{21}$$

2.6 Industrial output

Looking at the industrial sector in region *A*, the increasing economies of scale can be observed. Each company produces its own product, that is, there are many individual product variants, even though the companies are identical. This form of competition is called Chamberlain's monopolistic competition. This characteristic occurs only in the industrial sector. In addition, costs are incurred for the transportation of industrial goods. The partial mobility of the labor factor remains intact. The factor capital is completely mobile (Krugman, 1991a; 1991b).

The economies of scale are divided into external and internal returns to scale. The increasing internal economies of scale result from the unit costs of the companies. The production functions (*i.e., number of companies*) have a constant return on scale. The adaptation to increasing economies of scale takes place via the individual factor demand functions. Each individual commodity is produced under increasing economies of scale. The constant fixed costs and marginal costs lead to an increasing output (*due to decreasing average costs*).

Every company has fixed and marginal costs. The fixed and marginal costs are defined as F_M and c_M, respectively, to ensure increasing internal economies of scale. The substitution elasticity is defined as σ. The production costs, fixed costs, marginal costs, and elasticity of substitution are defined by equations (22)-(24):

$$\sigma > 1 \tag{22}$$

$$c_M < 1 \tag{23}$$

$$F_M > 0 \tag{24}$$

The industrial output of a company in advanced region *A* is defined by equation *(25)*. The individual output quantity of a company does not depend on productivity and the labor factor but on the elasticity of substitution, marginal costs, and fixed costs.

$$q_M = (\sigma - 1) \cdot \frac{F_M}{c_M} \tag{25}$$

The advanced region *A* specializes entirely in the comparative cost advantage and capital intensive technical equipment. The industrial workforce in region *A* is fully employed in the high-tech sector. Farmers in the agricultural sector are retraining and active in the industrial sector. In addition, the industrial workers of region *B* move to region *A* to work in the high-tech sector. The factor capital is mobile between regions and sectors. Returns are offset by factor migration. The industrial sector in region *A* is capital intensive, $\delta > \gamma$, and the agricultural sector in region *B* is labor intensive, $\gamma > \delta$. Overall, the output of industrial goods increases in relation to a one-factor model, such as the Ricardian model or core-peripheral model, due to capital mobility. Capital owners invest in region *A* (*advanced*). Region *B* produces fewer agricultural products due to the lower factors (*labor and capital*). This becomes clear when the individual factors are put into proportion. The factor demand functions are defined with:

$$l_M = F_M + c_M \cdot q_M \tag{26}$$

$$l_M = F_M + c_M \cdot (\sigma - 1) \cdot \frac{F_M}{c_M} = F_M + \sigma \cdot F_M - F_M = \sigma \cdot F_M \tag{27}$$

$$k_M = F_M + c_M \cdot q_M \tag{28}$$

$$k_M = F_M + c_M \cdot (\sigma - 1) \cdot \frac{F_M}{c_M} = F_M + \sigma \cdot F_M - F_M = \sigma \cdot F_M \tag{29}$$

The business-related factors are industrial labor l_m and industrial capital k_m, according to equations (27) and (29) (*new definition, i.e., not the available input factors labor and capital*). Input factors labor L_M and capital K_M influence the extended core-peripheral model via the number of companies' n. In this context, an artificial product unit is created to define the usage rate. Furthermore, region A represents the growing technology component $A_M > 1$.

$$n_M = \frac{A_M \cdot K_{M,SM}^{\delta} \cdot L_M^{1-\delta}}{k_M^{\delta} \cdot l_M^{1-\delta}} \tag{30}$$

$$n_M = \frac{A_M \cdot K_{M,SM}^{\delta} \cdot L_M^{1-\delta}}{(\sigma \cdot F_M)^{\delta} \cdot (\sigma \cdot F_M)^{1-\delta}} \tag{31}$$

The input factors do not affect the industrial quantity of an individual company but do affect the number of companies. The ratio of investment rate to depreciation rate defines the extent to which industrial output increases or decreases more sharply. Furthermore, a technology component leads to an increase in industrial output.

$$n_M = \frac{A_M \cdot \left((i_M - as_M) \cdot K_M\right)^{\delta} \cdot (L_M)^{1-\delta}}{(k_M)^{\delta} \cdot (l_M)^{1-\delta}} \tag{32}$$

$$n_M = \frac{A_M \cdot \left((i_M - as_M) \cdot K_M\right)^{\delta} \cdot (L_M)^{1-\delta}}{(\sigma \cdot F_M)^{\delta} \cdot (\sigma \cdot F_M)^{1-\delta}} \tag{33}$$

The total production is defined by multiplying the individual production quantity and the number of companies. The transportation costs T do not affect the industrial output.

$$Q_M = n_M \cdot q_M \tag{34}$$

2.7 Number of companies/distribution

We have already analyzed the number of companies in industrial output. The number of companies does not depend directly on the input factors.

$$n_M = \frac{A_M \cdot K_{M,SM}^{\delta} \cdot L_M^{1-\delta}}{k_M^{\delta} \cdot l_M^{1-\delta}} \tag{35}$$

$$n_M = \frac{A_M \cdot K_{M,SM}^{\delta} \cdot L_M^{1-\delta}}{(\sigma \cdot F_M)^{\delta} \cdot (\sigma \cdot F_M)^{1-\delta}} \tag{36}$$

The formal distribution of companies does not play a primary role in the model. The number of companies defines the distribution in a region. Due to the transportation costs, the distribution can also be considered later in the process via the core-peripheral model. This makes the model robust against extensions.

$$n_M = \frac{A_M \cdot ((i_M - as_M) \cdot K_M)^{\delta} \cdot (L_M)^{1-\delta}}{(k_M)^{\delta} \cdot (l_M)^{1-\delta}} \tag{37}$$

$$n_M = \frac{A_M \cdot ((i_M - as_M) \cdot K_M)^{\delta} \cdot (L_M)^{1-\delta}}{(\sigma \cdot F_M)^{\delta} \cdot (\sigma \cdot F_M)^{1-\delta}} \tag{38}$$

2.8 Cost functions

Cost function CO^* in the agricultural sector in region *B* is defined by the quantity produced, weighted by yields and wages.

$$CO_F^* = Q_F^* \cdot \left(w_F^* + r_F^*\right) \tag{39}$$

Cost function *CO* in the industrial sector in region *A* results from wages, returns, industrial labor, and industrial capital. The newly created product unit is defined as industrial factor costs.

$$CO_M = CO_{M,k} + CO_{M,l} \tag{40}$$

$$CO_M = (F_M + c_M \cdot q_M) \cdot (w_M + r_M) \tag{41}$$

The industrial input factors can also be considered individually.

$$CO_{M,k} = (F_M + c_M \cdot q_M) \cdot r_M \tag{42}$$

$$CO_{M,k} = \sigma \cdot F_M \cdot r_M \tag{43}$$

$$CO_{M,l} = \left(F_M + c_M \cdot q_M\right) \cdot w_M \tag{44}$$

$$CO_{M,l} = \sigma \cdot F_M \cdot w_M \tag{45}$$

2.9 Consumer benefits

Identical Cobb-Douglas utility functions are available in both regions. The respective utility function is defined in a two-stage decision-making process. The maximization of the first stage occurs between consumer goods and savings rates. In the second stage, the consumption of individual industrial goods is maximized and optimized. A special *CES* sub-utility function is available (Gumpert, 2016).

$$\max C_M = \left(\sum_{i=1}^{n} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{46}$$

The constraint for this is as follows:

$$\sum_i p_M \cdot c_i = \mu \cdot Y \quad \left(\equiv Y - C_A\right). \tag{47}$$

We define a world income *Y* for the connection between the regions. C_M indicates the consumption of industrial goods and C_F the consumption of agricultural goods. *S* also defines the savings rate.

$$Y = \left(C_M + S_M\right)^{\mu} \cdot \left(C_F + S_F\right)^{1-\mu} \tag{48}$$

$$\max U = S + C_M^{\mu} \cdot C_F^{1-\mu} \tag{49}$$

$$\max U = \left(C_M + \left(1 + i_M - as_M\right) \cdot Y\right)^{\mu} \cdot \left(C_F + \left(1 + i_F - as_F\right) \cdot Y\right)^{1-\mu} \tag{50}$$

These equations demonstrate the assumption that the number of companies can be infinite. The utility function has the special characteristic that a larger number of variations *i* increases the utility. Consumers maximize benefits while accounting for budget constraints, integrating a cost-cutting measure through the Solow model.

The total income results from the world market price multiplied by the total world production from all regions. The factor μ represents the share of capital goods expenditure in total world expenditure and the factor ($1 - \mu$) represents the share of food expenditure in total world expenditure.

$$P_M \cdot \left(Q_M + Q_M^{*}\right) = \mu \cdot Y \tag{51}$$

$$P_F \cdot \left(Q_F + Q_F^*\right) = (1-\mu) \cdot Y \tag{52}$$

2.10 Specialization pattern

For the pattern of specialization, the individual goods` prices P_M are put into proportion. The price of agricultural goods P_F is standardized to one. The price of industrial goods is then expressed in relation to the price of agricultural goods.

$$\frac{P_M}{P_F} = \frac{\mu}{1-\mu} \cdot \frac{Q_F + Q_F^*}{Q_M + Q_M^*} \tag{53}$$

After merging those equations *(21), (33), (34), (50), (51),* and *(52),* the following relation results:

$$\mu > \frac{A_F^* \cdot \left(\left(1 + i_F^* - as_F^*\right) \cdot K_F^*\right)^{\gamma} \cdot \left(L_F^*\right)^{1-\gamma}}{A_M \cdot \left(\left(1 + i_M - as_M\right) \cdot K_M\right)^{\delta} \cdot \left(L_M\right)^{1-\delta} \cdot \dfrac{\sigma}{\sigma - 1} \cdot c_M \cdot T} \tag{54}$$

2.11 Agricultural goods price

To simplify the model, the agricultural goods price is normalized to one. The industrial goods price is derived in relation to the price of agricultural goods.

2.12 Industrial goods price

The relative calculation of the industrial goods price in relation to the agricultural goods price is from the squared elasticity of substitution, the relative inequality, the maximum production quantities of the two regions with complete specializations, the squared marginal costs, and the transportation costs

$$P_M = \frac{\sigma}{\sigma - 1} \cdot \frac{\mu}{1-\mu} \cdot \frac{Q_F^*}{Q_M} \cdot c_M \cdot T \tag{55}$$

After observing the pattern of specialization and maximum production quantities, the ratio *(56)* is obtained. Within the modified input factors, the measures of the investment rates and depreciation rates of the Solow model also influence the present approach (Solow, 1956).

$$P_M = \left(\frac{\sigma}{\sigma - 1}\right)^2 \cdot \frac{\mu}{1-\mu} \cdot \frac{A_F^* \cdot \left(\left(1 + i_F^* - as_F^*\right) \cdot K_F^*\right)^{\gamma} \cdot \left(L_F^*\right)^{1-\gamma}}{A_M \cdot \left(\left(1 + i_M - as_M\right) \cdot K_M\right)^{\delta} \cdot \left(L_M\right)^{1-\delta}} \cdot c_M^2 \cdot T \tag{56}$$

Region *A* produces capital goods and region *B* produces agricultural goods. Region *A* is industrial and has not only its own labor force and capital resources but also a share of the two resources from the region *B*, characterized by its agricultural economy and low-incomes. The reduction of the agricultural products and the strengthening of industrial goods reduce the relative industrial product price. Furthermore, there is a greater variety of products and thus a higher benefit (*i.e., product variety*). The lower industrial commodity price correlates with the increase in industrial output volume. Region *B* provides fewer agricultural products, but these are more valuable.

Equation (57) shows that the input factor, productivity, and expenditure influence on the industrial goods price but as well as the substitution elasticity and cost function. The industrial goods price can also be considered for an industrial company (p_M).

$$p_M = \frac{\sigma}{\sigma - 1} \cdot \frac{\mu}{1 - \mu} \cdot \frac{Q_F^*}{q_M} \cdot c_M \cdot T \quad (57)$$

After the pattern of specialization has been determined and the maximum production quantities established, the following relation is obtained:

$$p_M = \frac{\sigma}{\sigma - 1} \cdot \frac{\mu}{1 - \mu} \cdot \frac{A_F^* \cdot \left(\left(1 + i_F^* - as_F^*\right) \cdot K_F^*\right)^{\gamma} \cdot \left(L_F^*\right)^{1-\gamma}}{(\sigma - 1) \cdot \frac{F_M}{c_M}} \cdot c_M \cdot T \quad (58)$$

2.13 Wages

Equations (59), (60), and (61) define wage in the industrial region *A*.

$$w_M = P_M \cdot T \cdot (\sigma - 1) \cdot \frac{F_M}{c_M} \cdot \frac{A_M}{\sigma \cdot F_M} \cdot (1 - \delta) \cdot \left(\frac{L_M}{\left(1 + i_M - as_M\right) \cdot K_M} \right)^{-\delta} \quad (59)$$

$$w_M = \frac{\sigma}{\sigma - 1} \cdot \frac{\mu}{1 - \mu} \cdot c_M \cdot T \cdot (1 - \delta) \cdot \frac{\left(\left(1 + i_F^* - as_F^*\right) \cdot K_F^*\right)^{\gamma} \cdot \left(L_F^*\right)^{1-\gamma}}{L_M} \quad (60)$$

$$w_M = \frac{\sigma}{\sigma - 1} \cdot \frac{\mu}{1 - \mu} \cdot \frac{\left(\left(1 + i_F^* - as_F^*\right) \cdot K_F^*\right)^{\gamma} \cdot \left(L_F^*\right)^{1-\gamma}}{L_M} \cdot c_M \cdot T \cdot (1 - \delta) \quad (61)$$

Equation *(62)* defines wage in the agricultural region *B*.

$$w_F^* = (1-\gamma)\cdot\left(\frac{L_F^*}{\left(1+i_F^*-as_F^*\right)\cdot K_F^*}\right)^{-\gamma} \tag{62}$$

Combining equations *(61)* and *(62)* yields the following:

$$\frac{w_M}{w_F^*} = \frac{\sigma}{\sigma-1}\cdot\frac{\mu}{1-\mu}\cdot\frac{1-\delta}{1-\gamma}\cdot\frac{L_F^*}{L_M}\cdot c_M\cdot T \tag{63}$$

Equation (63) shows that inequality between the two regions is influenced by monopolistic competition (*substitution elasticity* σ), partial mobility of labor and capital, marginal costs c_M, and transportation costs T. The substitution elasticity and transportation costs increase the wages of advanced region A. The partial-mobile labor input factor and fully mobile capital factor decrease the wage inequality between regions. The marginal costs reduce the wage gap.

2.14 Returns

Total income consists of wage and return. The returns for region A are as follows:

$$r_M = P_M\cdot\delta\cdot A_M\cdot\left(\frac{L_M}{\left(1+i_M-as_M\right)\cdot K_M}\right)^{1-\delta}\cdot T\cdot(\sigma-1)\cdot\frac{F_M}{c_M}\cdot\frac{1}{\sigma\cdot F_M} \tag{64}$$

$$r_M = \frac{\sigma}{\sigma-1}\cdot c_M\cdot T\cdot\frac{\mu}{1-\mu}\cdot\delta\cdot\frac{\left(\left(1+i_F^*-as_F^*\right)\cdot K_F^*\right)^{\gamma}\cdot\left(L_F^*\right)^{1-\gamma}}{\left(1+i_M-as_M\right)\cdot K_M} \tag{65}$$

Equation *(66)* defines return in the agricultural region B.

$$r_F^* = \gamma\cdot\left(\frac{L_F^*}{\left(1+i_F^*-as_F^*\right)\cdot K_F^*}\right)^{1-\gamma} \tag{66}$$

Combining equations *(65)* and *(66)* yields the following:

$$\frac{r_M}{r_F^*} = \frac{\sigma}{\sigma-1}\cdot c_M\cdot T\cdot\frac{\mu}{1-\mu}\cdot\frac{\delta}{\gamma}\cdot\frac{\left(1+i_F^*-as_F^*\right)\cdot K_F^*}{\left(1+i_M-as_M\right)\cdot K_M}. \tag{67}$$

The returns are identical across the regions. The relative capital stock is defined as follows:

$$\frac{\left(1+i_M-as_M\right)\cdot K_M}{\left(1+i_F^*-as_F^*\right)\cdot K_F^*}=\frac{\sigma}{\sigma-1}\cdot c_M\cdot T\cdot\frac{\mu}{1-\mu}\cdot\frac{\delta}{\gamma} \tag{68}$$

The assumption $K_M + K_F^* = 2$ or $K_M = 2 - K_F^*$, the substitution elasticity σ, the marginal costs c_M, the exponents of the production functions, and the exponents of the Cobb-Douglas utility function result in the optimal capital investment values and mobility decision (Gumpert, 2016).

$$K_F^*=\frac{2}{\frac{\sigma}{\sigma-1}\cdot c_M\cdot T\cdot\frac{\mu}{1-\mu}\cdot\frac{\delta}{\gamma}\cdot\frac{1+i_F^*-as_F^*}{1+i_M-as_M}+1}<1 \tag{69}$$

The input factor capital is defined as follows in the industrial sector of region *A*:

$$K_M=2-\frac{2}{\frac{\sigma}{\sigma-1}\cdot c_M\cdot T\cdot\frac{\mu}{1-\mu}\cdot\frac{\delta}{\gamma}\cdot\frac{1+i_F^*-as_F^*}{1+i_M-as_M}+1}>1 \tag{70}$$

Industrial technology is characterized by capital intensive production. Region *B*, by contrast, is labor intensive.

2.15 Incomes

In equations *(71)* and *(72)*, the incomes *EK* and EK^* in region *A* and region *B* are analyzed. Returns are balanced across regions and sectors by $r = r^*$.

$$EK=P_M\cdot Q_M-r_M\cdot\left(\left(1+i_M-as_M\right)\cdot K_M-1\right)-w_M\cdot\left(1-L_F^*\right) \tag{71}$$

$$EK^*=P_F^*\cdot Q_F^*+r_M\cdot\left(1-\left(1+i_F^*-as_F^*\right)\cdot K_F^*\right)+w_M\cdot\left(1-L_F^*\right) \tag{72}$$

In equations *(73)* and *(74)*, the prices and goods are resolved and explained in terms of content.

$$EK=\frac{\sigma}{\sigma-1}\cdot c_M\cdot T\cdot\frac{\mu}{1-\mu}\cdot A_F^*\cdot\left(\left(1+i_F^*-as_F^*\right)\cdot K_F^*\right)^{\gamma}\cdot\left(L_F^*\right)^{1-\gamma}-r_M\cdot\left(1+i_F^*-as_F^*\right)\cdot\left(K_M-1\right)-w_M\cdot L_M \tag{73}$$

$$EK^*=A_F^*\cdot\left(\left(1+i_F^*-as_F^*\right)\cdot K_F^*\right)^{\gamma}\cdot\left(L_F^*\right)^{1-\gamma}+r_F^*\cdot\left(1+i_F^*-as_F^*\right)\cdot\left(K_M-1\right)+w_M\cdot L_M \tag{74}$$

Clearly a low elasticity of substitution greatly increases the income of region *A*. Region *B* remains unchanged in terms of income. The same statement can be made for transportation costs, although transportation costs are not available as a net increase in income.

The marginal costs can also reduce and damage the income of advanced region *A* in relation to increasing learning effects. The reason for this is that industrial goods are produced in large quantities and agricultural goods are scarce and more valuable. Due to the higher volume of industrial goods, the relative price of the industrial product decreases. Finally, transfers of labor force and equalization of returns lead to a convergence of incomes.

3. Economic implications

The model combination results in a multitude of influencing factors for an active model application and further development:

- Rigid input factors have a negative effect on the harmonization of different living standards. An advanced region *A* can protect itself from low-wage workers; a backward region *B* can hinder development. The factor can be divided n-fold. Furthermore, a binding, for example, from the labor factor to the soil factor, leads to a simulation of fixed spatial factors.
- Expansion to an additional mobile factor such as capital simulates the possibility of balancing marginal returns and integrates globalization into the model. As a result, the regions converge.
- Scale effects can be defined internally and externally as falling, constant, and increasing.
- The patterns of specialization can be selected according to the comparative advantage, factor endowments, learning effect, or growth theoretical approach. Due to the robustness of the model, identical patterns of specializations will always be available.
- Different market forms are combined and can be analyzed. In the advanced region *A*, monopolistic competition was established and characterized by product diversity. In the region *B*, perfect competition has been allowed.
- The model allows analyses of cost forms while marginal costs have an increasing effect on income. By contrast, elasticity of substitution leads to an increase in income decline. Furthermore, the utility function can compare two external products (i.e., agricultural and industrial goods) and internally reflect product diversity and product variants for industrial goods.
- Transportation costs simulate an effort to move industrial goods, and this represents a room component. Transportation costs make industrial goods less attractive and more expensive.
- Consider the investment ratio, depreciation rates (*"steady state"*), and savings ratios from the Solow model. A higher investment rate than depreciation ratio allows region *A* to develop. The direct link between

consumers (*savers*) and companies (*producers*) via the investment saving rate reduces consumption in region *A*.

- The dynamic technology component occurs through capital write-downs and learning effects. A rising capital stock increases the learning experience and technology component.
- Finally, a standard unit will be introduced in the industrial sector to allow for a direct link between the number of enterprises and the individual production quantities of an individual enterprise. This allowance enables a *”bridge”* to the out analysis of complete competition. Furthermore, spatial aspects can be integrated.

4. Conclusion

This paper presents a combination model created from multiple central economic models, such as Ricardian, Heckscher-Ohlin, Krugman, or Solow. The combined model shows a high level of robustness; furthermore, it allows specialization according to comparative advantage, factor endowments or growth theoretical mechanisms without being contradictory in itself. The model allows for a variety of analysis options because of the many combined influencing factors. The aim of this publication is to contribute to creating a more robust model framework and the ability to offer and conduct broader analyses.

References:

Boschma, R., Frenken, K. 2017. Evolutionary Economic Geography. The New Oxford Handbook of Economic Geography. Oxford University Press, 213-242.

Bretschger, L. 2004. Growth theory, 3rd Edition. Oldenbourg Publishing House.

Brezis, E.S., Krugman, P.R., Tsiddon, D. 1993. Leapfrogging in International Competition: A Theory of Cycles in National Technological Leadership. American Economic Review, 83, 1211-1219.

Brezis, E.S., Tsiddon, D. 1998. Economic Growth, Leadership and Capital Flows: The Leapfrogging Effect. Journal of International Trade and Economic Development, 7(3), 261-277.

Cobb, C.W., Douglas, P.H. 1928. A Theory of Production. American Economic Review, Supplement, 18(1), 139-165.

Demko, G. 2017. Regional Development: Problems and Policies in Eastern and Western Europe. Routledge Library Editions: Urban and regional Economics, 68.

Desmet, K., Ortín, I.O. 2007. Rational Underdevelopment. Scandinavian Journal of Economics, 109, 1-24.

Dixit, A., Stiglitz, J. 1977. Monopolistic Competition and Optimum Product Diversity. American Economic Review, 76(3), 297-308.

Ehrenfeld, W. 2004. The New Economic Geography. Munich Archive, Universität Regensburg, MPRA Paper No. 12232.

Fonseca, M. 2017. Southern Europe at a Glance: Regional Disparities and Human Capital. Regional Upgrading in Southern Europe, Springer Nature Publishing, 19-54.

Gärtner, M. 2006. Macroeconomics, 2nd Edition. Pearson Education.

Gumpert, M. 2013. Rational underdevelopment - uneven development in open economies in connection with financial transfers, learning externalities and different technologies - analytical analysis under different model frameworks. RUFIS Institute, University publisher Brockmeyer.

Gumpert, M. 2016. Rational underdevelopment: regional economic disparities under the Heckscher-Ohlin Theorem. International Review of Applied Economics, 30(1), Routledge Taylor & Francis Group.

Heinemann, M. 2015. Dynamic Macroeconomics. The Ramsey-Model, Springer Gabler.

Krugman, P. 1991a. Geography and Trade. Cambridge, JPE.

Krugman, P. 1991b. Increasing Returns and Economic Geography. Journal of Political Economy, 99(3), 483-499.

Krugman, P., Obstfeld, M., Melitz, M. 2011. International Economics: Theory und Policy, 9th Edition. Addison-Wesley publisher, Global Edition, Upper Saddle River.

Krugman, P., Venables, A.J. 1990. Integration and the competitiveness of the peripheral industry. In Bliss, C., Braga De Macedo, J. (eds.) 1990, Unity with Diversity in the European Economy. Cambridge/London: Cambridge University Press/CEPR, 55-77.

Krugman, P., Venables, A. 1995. Globalization and the Inequalities of Nations. Quarterly Journal of Economics, 110.

Martin, R., Sunley, P. 2017. Paul Krugman's Geographical Economics and Its Implications for Regional Development Theory: A Critical Assessment. Taylor & Francis Group, Chapter 2.

Ohlin, B. 1933. Interregional and International Trade. Cambridge, MA: Harvard University Press.

Pindyck, R.S., Rubinfeld, D.L. 2003. Microeconomics, 5th Edition, Pearson Studies.

Ramsey, F.P. 1928. A mathematical theory of saving. The Economic Journal, 543-559.

Ricardo, D. 1987. On the Principles of Political Economy and Taxation. Dent, London, Melbourne, 84.

Samuelson, P.A., Nordhaus, W.D. 2005. Macroeconomics-The international standard work of macro-and microeconomics. Redline GmbH.

Solow, R.M. 1956. A Contribution to the Theory of Economic Growth. Quarterly Journal of Economics, 70, 65-94.

Swan, T. 2004. Economic Growth and Capital Accumulation. Economic Record, Volume 32, Issue 2, 334-361.

Venables, A. 1996. Equilibrium Locations of Vertically Linked Industries. International Economic Review, 37, 341-359.